Summer Math Assignment

Resource Geometry

Directions: Please respond to the following questions in this packet.
Using a Ruler and Protractor

Knowing how to use a ruler and protractor is crucial for success in geometry.

Example

Draw a triangle that has a $28^\circ$ angle between sides of length 5.2 cm and 3.0 cm.

Step 1: Use a ruler to draw a segment 5.2 cm long. 

Step 2: Place the hole of a protractor at one endpoint of the segment. Make a small mark at the $28^\circ$ position along the protractor.

Step 3: Align the ruler along the small mark and the same endpoint. Place the zero point of the ruler at the endpoint. Draw a segment 3.0 cm long.

Step 4: Complete the triangle by connecting the endpoints of the first and second segments.

Exercises

1. Measure sides $AB$ and $BC$ of $\triangle ABC$ to the nearest millimeter.

2. Measure each angle of $\triangle ABC$ to the nearest degree.

3. Draw a triangle that has a side of length 2.4 cm between a $43^\circ$ angle and a $102^\circ$ angle.
Classifying Triangles

You can classify a triangle by its angles and sides.

- **Equiangular**: all angles congruent
- **Acute**: all angles acute
- **Right**: one right angle
- **Obtuse**: one obtuse angle

- **Equilateral**: all sides congruent
- **Isosceles**: at least two sides congruent
- **Scalene**: no sides congruent

**Example**

What type of triangle is shown below?

![Triangle](image)

At least two sides are congruent, so the triangle is isosceles. One angle is obtuse, so the triangle is obtuse. The triangle is an obtuse isosceles triangle.

**Exercises**

Classify each triangle by its sides and angles.

1. ![Triangle](image)
2. ![Triangle](image)
3. ![Triangle](image)

If possible, draw a triangle to fit each description. Mark the triangle to show known information. If you cannot draw the triangle, write *not possible* and explain why.

4. acute equilateral
5. right equilateral
6. obtuse scalene
7. acute isosceles
8. right isosceles
9. acute scalene
Measurement Conversions

To convert from one unit of measure to another, you multiply by a conversion factor in the form of a fraction. The numerator and denominator are in different units, but they represent the same amount. So you can think of this as multiplying by 1.

An example of a conversion factor is \( \frac{1\text{ ft}}{12\text{ in.}} \). You can create other conversion factors using the table on page 837.

**Example 1**

Complete each statement.

a. \( 88\text{ in.} = \_\text{ ft} \)
   \[
   88\text{ in.} \cdot \frac{1\text{ ft}}{12\text{ in.}} = \frac{88}{12}\text{ ft} = 7\frac{1}{3}\text{ ft}
   \]

b. \( 5.3\text{ m} = \_\text{ cm} \)
   \[
   5.3\text{ m} \cdot \frac{100\text{ cm}}{1\text{ m}} = 5.3(100)\text{ cm} = 530\text{ cm}
   \]

Area is always in square units, and volume is always in cubic units.

\[
\begin{align*}
3\text{ ft} & \hspace{1cm} 3\text{ ft} \\
3\text{ ft} & \hspace{1cm} 3\text{ ft} \\
3\text{ ft} & \hspace{1cm} 3\text{ ft}
\end{align*}
\]

1 yd = 3 ft \hspace{1cm} 1 yd^2 = 9 ft^2 \hspace{1cm} 1 yd^3 = 27 ft^3

**Example 2**

Complete each statement.

a. \( 300\text{ in.}^2 = \_\text{ ft}^2 \)
   \[
   1\text{ ft} = 12\text{ in.}, \text{ so } 1\text{ ft}^2 = (12\text{ in.})^2 = 144\text{ in.}^2.
   \]
   \[
   300\text{ in.}^2 \cdot \frac{1\text{ ft}^2}{144\text{ in.}^2} = 2\frac{1}{12}\text{ ft}^2
   \]

b. \( 200,000\text{ cm}^3 = \_\text{ m}^3 \)
   \[
   1\text{ m} = 100\text{ cm}, \text{ so } 1\text{ m}^3 = (100\text{ cm})^3 = 1,000,000\text{ cm}^3.
   \]
   \[
   200,000\text{ cm}^3 \cdot \frac{1\text{ m}^3}{1,000,000\text{ cm}^3} = 0.2\text{ m}^3
   \]

**Exercises**

Complete each statement.

1. \( 40\text{ cm} = \_\text{ m} \)
2. \( 1.5\text{ kg} = \_\text{ g} \)
3. \( 60\text{ cm} = \_\text{ mm} \)
4. \( 200\text{ in.} = \_\text{ ft} \)
Squaring Numbers and Finding Square Roots

The square of a number is found by multiplying the number by itself. An exponent of 2 is used to indicate that a number is being squared.

**Example 1**

Simplify.

a. \(5^2\)
   
   \[5^2 = 5 \cdot 5 = 25\]

b. \((-3.5)^2\)
   
   \[(-3.5)^2 = (-3.5) \cdot (-3.5) = 12.25\]

c. \((\frac{2}{7})^2\)
   
   \[\left(\frac{2}{7}\right)^2 = \frac{2}{7} \cdot \frac{2}{7} = \frac{4}{49}\]

The square root of a number is itself a number that, when squared, results in the original number. A radical symbol \((\sqrt{\cdot})\) is used to represent the positive square root of a number.

**Example 2**

Simplify. Round to the nearest tenth if necessary.

a. \(\sqrt{36}\)
   
   \[\sqrt{36} = 6, \text{ since } 6^2 = 36.\]

b. \(\sqrt{174}\)
   
   \[\sqrt{174} \approx 13.2, \text{ since } 13.2^2 \approx 174.\]

You can solve equations that include squared numbers.

**Example 3**

**Algebra** Solve.

a. \(x^2 = 144\)
   
   \[x = 12 \text{ or } -12\]

b. \(a^2 + 3^2 = 5^2\)
   
   \[a^2 + 9 = 25\]

   \[a^2 = 16\]

   \[a = 4 \text{ or } -4\]

**Exercises**

Simplify.

1. \(11^2\)

2. \((-14)^2\)

Simplify. Round to the nearest tenth if necessary.

7. \(\sqrt{100}\)

8. \(\sqrt{169}\)

**Algebra** Solve. Round to the nearest tenth if necessary.

13. \(x^2 = 49\)

17. \(a^2 + b^2 = 10^2\)
Evaluating and Simplifying Expressions

To evaluate an expression with variables, substitute a number for each variable. Then simplify the expression using the order of operations. Be especially careful with exponents and negative signs. For example, the expression \(-x^2\) always yields a negative or zero value, and \((−x)^2\) is always positive or zero.

**Example 1**

**Algebra** Evaluate each expression for \(r = 4\).

a. \(-r^2\)
   \[-r^2 = -4^2 = -16\]

b. \(-3r^2\)
   \[-3r^2 = -3(4^2) = -3(16) = -48\]

c. \((r + 2)^2\)
   \[(r + 2)^2 = (4 + 2)^2 = (6)^2 = 36\]

To simplify an expression, you eliminate any parentheses and combine like terms.

**Example 2**

**Algebra** Simplify each expression.

a. \(5r - 2r + 1\)
   
   Combine like terms.
   \[5r - 2r + 1 = 3r + 1\]

b. \(\pi(3r - 1)\)
   
   Use the Distributive Property.
   \[\pi(3r - 1) = 3\pi r - \pi\]

c. \((r + \pi)(r - \pi)\)
   
   Multiply polynomials.
   \[(r + \pi)(r - \pi) = r^2 - \pi^2\]

**Exercises**

**Algebra** Evaluate each expression for \(x = 5\) and \(y = -3\).

1. \(-2x^2\)
2. \(-y + x\)
3. \(-xy\)

**Algebra** Simplify.

17. \(6x - 4x + 8 - 5\)
18. \(2(\ell + \omega)\)
19. \(-(4x + 7)\)
Simplifying Ratios

The ratio of the length of the shorter leg to the length of the longer leg for this right triangle is 4 to 6. This ratio can be written in three ways.

\[ \frac{4}{6} \quad 4 : 6 \]

Example

Algebra  Simplify each ratio.

a. 4 to 6

\[
4 \text{ to } 6 = \frac{4}{6} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3}
\]

b. 3ab : 27ab

\[
3ab : 27ab = \frac{3ab}{27ab} = \frac{3ab}{3ab} \cdot \frac{1}{9} = \frac{1}{9}
\]

c. \(\frac{4a + 4b}{a + b}\)

\[
\frac{4a + 4b}{a + b} = \frac{4(a + b)}{a + b} = 4
\]

Exercises

Algebra  Simplify each ratio.

1. 25 to 15
2. 6 : 9
3. \(\frac{36}{54}\)

Express each ratio in simplest form.

19. shorter leg : longer leg

20. hypotenuse to shorter leg
Absolute Value

Absolute value is used to represent the distance of a number from 0 on a number line. Since distance is always referred to as a nonnegative number, the absolute value of an expression is nonnegative.

On the number line at the right, both 4 and −4 are four units from zero. Therefore, |4| and |−4| are both equal to four.

When working with more complicated expressions, always remember to simplify within absolute value symbols first.

**Example 1**

Simplify each expression.

a. |4| + |−19|  
   |4| + |−19| = 4 + 19  
   = 23

b. |4 − 8|  
   |4 − 8| = |−4|  
   = 4

c. −3|−7 − 4|  
   −3|−7 − 4| = −3|−11|  
   = −3·11  
   = −33

To solve the absolute value equation |x| = a, find all the values x that are a units from 0 on a number line.

**Example 2**

**Algebra** Solve.

a. |x| = 7  
   x = 7 or −7

b. |x| − 3 = 22  
   |x| = 25  
   x = 25 or −25

**Exercises**

Simplify each expression.

1. |−8|  
2. |11|  
3. |−7| + |15|

**Algebra** Solve.

13. |x| = 16  
14. 1 = |x|  
15. |x| + 7 = 27
The Coordinate Plane

Two number lines that intersect at right angles form a coordinate plane. The horizontal axis is the x-axis and the vertical axis is the y-axis. The axes intersect at the origin and divide the coordinate plane into four sections called quadrants.

An ordered pair of numbers names the location of a point in the plane. These numbers are the coordinates of the point. Point B has coordinates (−3, 4).

You use the x-coordinate to tell how far to move right (positive) or left (negative) from the origin. You then use the y-coordinate to tell how far to move up (positive) or down (negative) to reach the point (x, y).

Example 1

Graph each point in the coordinate plane. In which quadrant or on which axis would you find each point?

a. Graph point A(−2, 3) in the coordinate plane.
   To graph A(−2, 3), move 2 units to the left of the origin. Then move 3 units up. Since the x-coordinate is negative and the y-coordinate is positive, point A is in Quadrant II.

b. Graph point B(2, 0) in the coordinate plane.
   To graph B(2, 0), move 2 units to the right of the origin. Since the y-coordinate is 0, point B is on the x-axis.

Exercises

Name the coordinates of each point in the coordinate plane at the right.

1. S

Graph each ordered pair in the same coordinate plane.

5. (0, −5)

In which quadrant or on which axis would you find each point?

9. (0, 10)
Solving and Writing Linear Equations

To solve a linear equation, use the properties of equality and properties of real numbers to find the value of the variable that satisfies the equation.

**Example 1**

**Algebra** Solve each equation.

1. \(5x - 3 = 2\)
   - \(5x = 5\) Add 3 to each side.
   - \(x = 1\) Divide each side by 5.

2. \(1 - 2(x + 1) = x\)
   - \(1 - 2x - 2 = x\) Use the Distributive Property.
   - \(-1 - 2x = x\) Simplify the left side.
   - \(-1 = 3x\) Add 2x to each side.
   - \(\frac{1}{3} = x\) Divide each side by 3.

You will sometimes need to translate word problems into equations. Look for words that suggest a relationship or some type of mathematical operation.

**Example 2**

**Algebra** A student has grades of 80, 65, 78, and 92 on four tests. What is the minimum grade she must earn on her next test to ensure an average of 80?

**Relate** average of 80, 65, 78, 92, and next test is 80

**Define** Let \(x\) = the grade on the next test.

**Write** \(\frac{80 + 65 + 78 + 92 + x}{5} = 80\)

- \(\frac{315 + x}{5} = 80\) Combine like terms.
- \(315 + x = 400\) Multiply each side by 5.
- \(x = 85\) Subtract 315 from each side.

The student must earn 85 on the next test for an average of 80.

**Exercises**

**Algebra** Solve each equation.

1. \(3n + 2 = 17\)
2. \(5a - 2 = -12\)
3. \(2x + 4 = 10\)

13. Twice a number subtracted from 35 is 9. What is the number?
Percents

A percent is a ratio in which a number is compared to 100. For example, the expression 60 percent means “60 out of 100.” The symbol % stands for “percent.”

A percent can be written in decimal form by first writing it in ratio form, and then writing the ratio as a decimal. For example, 25% is equal to the ratio \( \frac{25}{100} \) or \( \frac{1}{4} \). As a decimal, \( \frac{1}{4} \) is equal to 0.25. Note that 25% can also be written directly as a decimal by moving the decimal point two places to the left.

**Example 1**
Convert each percent to a decimal.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 42%</td>
<td>b. 157%</td>
<td>c. 12.4%</td>
<td>d. 4%</td>
</tr>
<tr>
<td>42% = 0.42</td>
<td>157% = 1.57</td>
<td>12.4% = 0.124</td>
<td>4% = 0.04</td>
</tr>
</tbody>
</table>

To calculate a percent of a number, write the percent as a decimal and multiply.

**Example 2**
Simplify. Where necessary, round to the nearest tenth.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>a. 30% of 242</td>
<td>b. 7% of 38</td>
</tr>
<tr>
<td>30% of 242 = 0.3 \times 242</td>
<td>7% of 38 = 0.07 \times 38</td>
</tr>
<tr>
<td>= 72.6</td>
<td>= 2.66 ≈ 2.7</td>
</tr>
</tbody>
</table>

For a percent problem, it is a good idea to check that your answer is reasonable by estimating it.

**Example 3**
Estimate 23% of 96.

23% ≈ 25% and 96 ≈ 100. So 25% \( \left( or \frac{1}{4} \right) \) of 100 = 25.
A reasonable estimate is 25.

**Exercises**

Convert each percent to a decimal.

1. 50%
2. 27%

Simplify. Where necessary, round to the nearest tenth.

7. 21% of 40

Estimate.

11. 12% of 70